Calculation of the Unknown Criteria Weights Under the Neutrosophic TOPSIS and the Neutrosophic VIKOR MCGDM Problems

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Abstract— This paper presents three different proposed methods for determining the suitable criteria weights with the nature of being completely unknown or partially known. These weights are applied to the Neutrosophic TOPSIS and the Neutrosophic VIKOR separately for solving the Multi Criteria Group Decision Making (MCGDM) problems and ranking the alternatives. In the first proposed technique, the entropy weights method is used for calculating the completely unknown criteria weights. In the second proposed technique, the maximizing deviation method is concerned with the calculation of the criteria weights either completely unknown or partially known. A linear programming model is constructed for the partially known case while a non-linear programming model is constructed for the completely unknown case. The third technique proposed two different multi-objectives linear programming models for calculating the partially known criteria weights. For calculating the completely unknown criteria weights, two different multi-objectives non-linear programming models are constructed. Finally, all the proposed methods are applied to a numerical example and compared with each other to justify their applicability and effectiveness. The effect of the computation time is studied to recommend the most suitable method in calculating the criteria weights.

Index Terms— Entropy weights, Maximuzing deviation, MCGDM, Neutrosophic, Optimization, TOPSIS, VIKOR.

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1 INTRODUCTION

Solving the MCGDM problems is a major challenge to the researchers especially under the uncertainty cases. To handle the data uncertainty in this type of problems, Zadeh [1] firstly introduced the fuzzy sets (FS) which characterized by a membership function ranging from 0 to 1. Then, Atanassov [2] introduced the intuitionistic fuzzy sets (IFS) to overcome the shortage of the FS to handle the non-membership function. Neutrosophic sets (NS) proposed by Smarandache [3] to deal with the inability of the IFS to handle the intermediate and inconsistent information. It was found that NS is very difficult to be applied to the real-life situations, so Wang [4] introduced the single-valued Neutrosophic sets (SVNs) to be applied to the real problems.

Some of the crisp techniques for solving the MCGDM problems have been extended to handle the problems with Neutrosophic information such as COPRAS [5], GRA [6], SWARA [7], ELECTRE [8], MULTIMOORA [9], and PROMETHEE [10]. Another technique is the Neutrosophic technique for order preference by similarity to ideal solution (TOPSIS) presented by Biswas [11]. A general view of the single-valued Neutrosophic TOPSIS (SVN-TOPSIS) was provided in [12]. Neutrosophic TOPSIS has been extended to handle the MCGDM problems with interval-valued Neutrosophic sets in [13] for the selection of a company. Neutrosophic TOPSIS has been applied in major number of researches for solving different MCDM problems. In [14], it is applied for solving the supplier selection problem. In [15] the researchers tried to simplify the calculations of the Neutrosophic TOPSIS.

Another technique for solving the Neutrosophic MCDM problems is the VIKOR method presented in [16] applied for the location selection problem. The Neutrosophic VIKOR has been extended for the interval Neutrosophic group decision making problems as in [17]. Tan et al. [18] applied the Neutrosophic VIKOR for the emergency decision making problems. Neutrosophic TOPSIS and VIKOR applied together in [19] to solve the Neutrosophic MCGDM problem with single-valued Neutrosophic numbers.

The criteria weights in the MCGDM problems may be of a completely unknown or a partially known nature. These weights can be calculated by entropy method, maximizing deviation method, or optimization method. The entropy is used to measure the uncertainty degree exists in a system. Entropy method is applied for solving the Neutrosophic decision making problems as in [20, 21, 22]. The maximizing deviation method developed by Wang in [23] assigns the criteria with larger deviation, a larger weight and vice versa. The maximizing deviation method is used with neutrosophic information to find the unknown criteria weights by Şahin [24]. The optimization method based on finding the weights by using the linear and nonlinear mathematical models. Biswas et al. uses the optimization method in solving the decision-making problems in [6].

The main problem faced the researchers in solving the MCGDM problems is the calculation of the criteria weights. The main objective of this paper is calculating the unknown criteria weights either in the completely unknown or the partially known cases using three different proposed techniques. After calculating these weights, they are applied to the Neutrosophic TOPSIS and to the Neutrosophic VIKOR methods separately for solving the MCGDM problem and ranking the given alternatives. The first proposed technique can be applied to calculate the completely unknown criteria weights using the entropy weights method. In the second proposed technique, the maximizing deviation method is used to calculate the weights by constructing two different mathematical models. Multi-objectives models are constructed in the third proposed technique for finding the unknown weights based on the Neutrosophic TOPSIS and the Neutrosophic VIKOR. For the partially known case, two different multi-objectives linear programming models are

proposed. Whereas, in the completely unknown case, two different multi-objectives non-linear programming models are proposed for calculating the weights. The complexity of calculations in the three proposed techniques is then studied with respect to the computation time to determine the most suitable one.

The remainder of this paper is organized as follows: Section. 2 briefly introduces some preliminaries related to this research. Section. 3 presents two different methods for solving the Neutrosophic MCGDM problems, TOPSIS and VIKOR. Section. 4 highlights the proposed algorithms for calculating the unknown criteria weights and solving the MCGDM problem. A numerical example is conducted to illustrate the proposed methods in Section. 5. The results and discussions of the proposed techniques are conducted in Section. 6. Finally, conclusions and future work are pointed out at Section. 7.

2 PRELIMINARIES

In this section, a brief review of the basic concepts and properties of Neutrosophic sets have been provided to be used in this paper

Neutrosophic Set

Neutrosophic set is a part of Neutrosophic which is a new branch of philosophy introduced by Smarandache in [3].

Single Valued Neutrosophic Set (SVNs)

The Single-valued Neutrosophic set is a special subclass of the Neutrosophic set to handle the real-life problems. Some basic definitions of SVNs are given in [4].

Hamming Distance

The hamming distance is used as a tool to measure the distance between any two Neutrosophic sets.

Definition 1. [6, 20] Let $a_1 = (T_1, I_1, F_1)$, and $a_2 = (T_2, I_2, F_2)$ be two SVNs, then the hamming distance between the two sets a_1, a_2 is:

$$D_H(a_1, a_2) = \sum_{i=1}^{n}(|T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2|) \qquad (1)$$

and the normalized hamming distance between any two SVNs is:

$$D_{NH}(a_1, a_2) = \frac{1}{3n}\sum_{i=1}^{n}(|T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2|) \qquad (2)$$

Euclidean Distance

The Euclidean distance is used to measure the separation distance between any two SVNs.

Definition 2. [19] Let $a_1 = (T_1, I_1, F_1)$, and $a_2 = (T_2, I_2, F_2)$ be two SVNs, then the Euclidean distance between the two sets a_1, a_2 is:

$$d(a_1, a_2) = \sqrt{\frac{1}{3}\sum_{i=1}^{n}(|T_1 - T_2|)^2 + (|I_1 - I_2|)^2 + |F_1 - F_2|^2} \qquad (3)$$

and the normalized Euclidean distance between any two SVNs is:

$$d(a_1, a_2) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}(|T_1 - T_2|)^2 + (|I_1 - I_2|)^2 + |F_1 - F_2|^2} \qquad (4)$$

Single-Valued Neutrosophic Weighted Average Operator (SVNWA)

Let $a = (T_j, I_j, F_j)$, $(j = 1,2, ..., n)$ be a collection of SVN numbers, then the SVNWA operator is defined as:

Definition 3. [19] $SVNWA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j a_j$
where w_j is the weight of $a_j(1,2, ..., n)$, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$

$$SVNWA = \langle 1 - \prod_{j=1}^{n}(1 - T_{ij})^{w_j}, \prod_{j=1}^{n}(I_{ij})^{w_j}, \prod_{j=1}^{n}(F_{ij})^{w_j} \rangle \qquad (5)$$

3 NEUTROSOPHIC METHODS FOR SOLVING THE MCGDM PROBLEMS

In this section, a brief review of the Neutrosophic TOPSIS and the Neutrosophic VIKOR is introduced. Consider a multi-criteria group decision-making problem with m alternatives, n criteria, and p decision makers.

Let $K = \{K_1, K_2, ..., K_p\}$ be a decision-making group
$A = \{A_1, A_2, ..., A_m\}$ be a set of alternatives
$C = \{C_1, C_2, ..., C_n\}$ be a set of criteria
$W = \{W_1, W_2, ..., W_n\}$be the set of criteria weights which may be completely unknown or partially known such that, $\Sigma W_j = 1$, $W_j \geq 0$, $j = (1, 2,..., n)$
$D = \{d_{ij}\}, i = 1,2, ..., m, \ j = 1,2, ..., n$ be the performance ratings of alternatives with respect to each criterion. These values are recorded in a decision matrix like the following one for each decision maker.

$$D = (d_{ij})_{m \times n} = \begin{array}{c} A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{pmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{pmatrix}$$

3.1 Neutrosophic TOPSIS

The concept of the traditional TOPSIS was extended to solve the Neutrosophic MCGDM problems, where all the ratings of alternatives are in the form of Neutrosophic sets. Single-valued Neutrosophic TOPSIS uses Single-valued Neutrosophic sets to determine the performance ratings. Where, interval-valued Neutrosophic sets are used in the interval-valued Neutrosophic TOPSIS. Ranking the alternatives is depending on the concept that, the most suitable one is the nearest to the Neutrosophic Positive Ideal Solution (SVN-PIS) and the farthest from the Neutrosophic Negative Ideal Solution (SVN-NIS). The main steps of the Neutrosophic TOPSIS as aforementioned in [11] by Biswas.

3.2 Neutrosophic VIKOR

The Neutrosophic VIKOR has been extended to deal with the decision problems of Neutrosophic nature with conflicting and non-commensurable criteria. VIKOR's main concept based on determining a compromise solution which is the closest to the ideal solution. This compromise solution has the maximum group utility for the majority and has the minimum of individual regret for the opponent. The main steps summarized the Neutrosophic VIKOR as in [19].

4 THE PROPOSED TECHNIQUES

The main challenge in solving the MCGDM problems, is to determine the unknown criteria weights even if they are completely unknown or partially known. In this paper, three different proposed techniques are introduced to calculate these weights. After calculating the unknown weights, the alternatives in the MCGDM problem are ranked by two different

methods, Neutrosophic TOPSIS and Neutrosophic VIKOR sep-
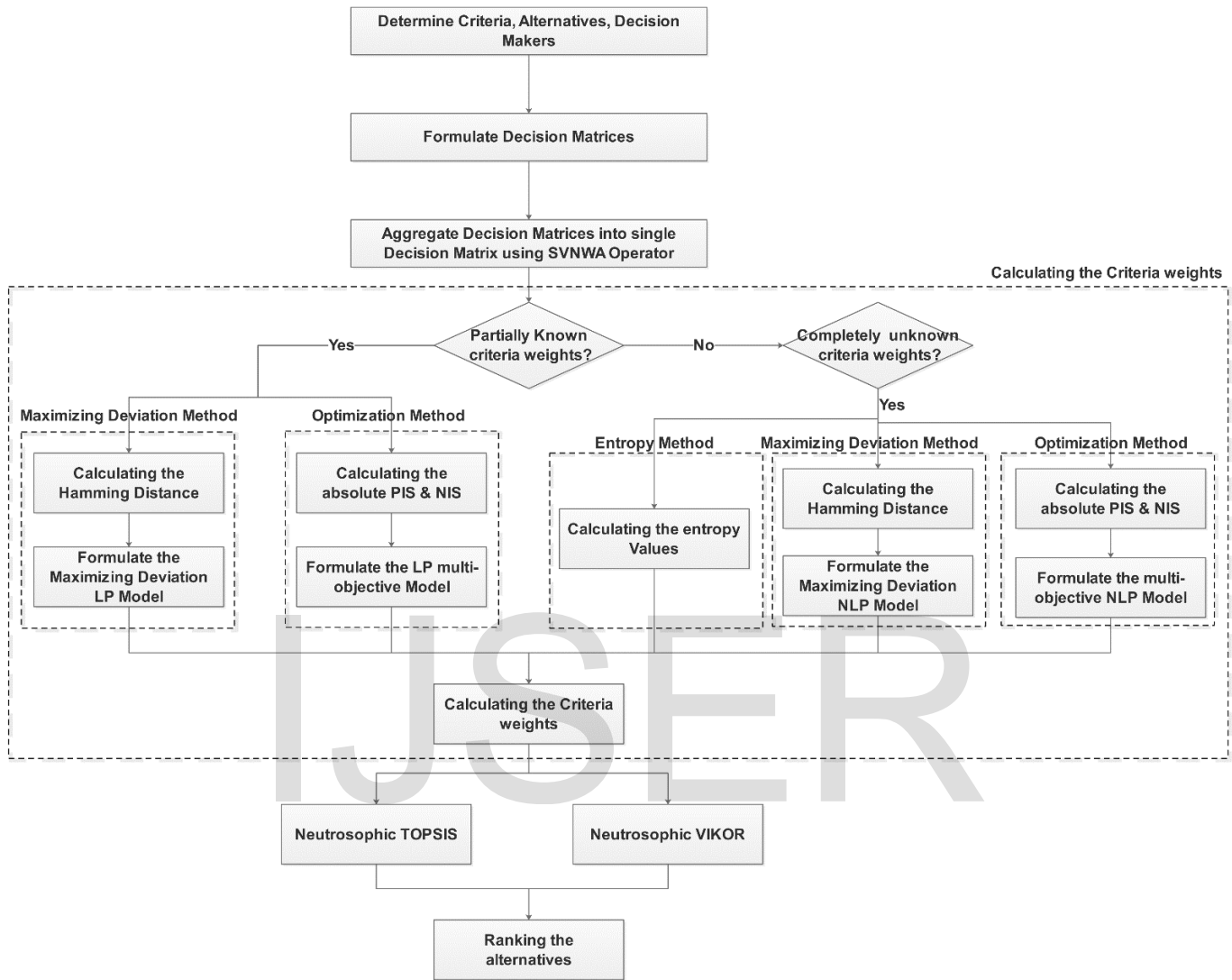arately. The proposed techniques are illustrated in Fig. (1).

Fig. 1. Flowchart of the proposed method.

4.1 The First Proposed Technique (using the entropy weights):

This technique depends on using the entropy weights method
discussed in [20] to calculate the completely unknown weights
for criteria. The main steps of this technique are as follows:

i. Aggregation of the decision matrices into a single decision matrix:

Each decision maker has his own decision matrix, that are com-
bined by using the SVNWA operator to formulate a separate
decision matrix by (6).

$$d_{ij} = \left(1 - \prod_{K=1}^{p}\left(1 - T_{ij}\right)^{\lambda_K}, \prod_{K=1}^{p}\left(I_{ij}\right)^{\lambda_K}, \prod_{K=1}^{p}\left(F_{ij}\right)^{\lambda_K}\right) \quad (6)$$

where λ_K is the weight of the decision maker

ii. Calculation of the entropy values:

By using (7), the value of the entropy (E_j) for each criterion is
calculated as

$$E_j = 1 - \frac{1}{n}\sum_{i=1}^{m}\left(T_{ij}(X_i) + F_{ij}(X_i)\right)\left|I_{ij}(X_i) - I_{ij}^c(X_i)\right| \quad (7)$$

iii. Calculation of the criteria weights:

The criteria weights are then calculated depending on the val-
ues of the entropy by (8)

$$W_j = \frac{1 - E_j}{\sum_{j=1}^{n} 1 - E_j} \quad (8)$$

After that, these weights are applied once to the TOPSIS and
again to the VIKOR for solving the decision-making problem
and ranking the alternatives.

4.2 The second proposed technique (using the maximizing deviation method):

This method depends on using the maximizing deviation

method for calculating the criteria weights. Firstly, all the decision matrices are aggregated into single decision matrix using the SVNWA operator in (6). Then, a linear programming model is applied for the case of the partially known criteria weights (9) and a non-linear programming model is applied for the case of the completely unknown criteria weights (10). After that, TOPSIS and VIKOR are applied separately for solving the MCGDM problem and ranking the alternatives.

Partially known criteria weights:
The weights are calculated per the following linear programming model:

$$\begin{cases} \max \mathrm{H(w)} = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{r=1}^{m} D_{NH}(\tilde{a}_{ij}, \tilde{a}_{rj})W_j \\ \mathrm{s.t.} \\ W_j \geq 0, \sum_{j=1}^{n} W_j = 1, j = 1,2,\cdots,n \end{cases} \quad (9)$$

Where, D_{NH} is the normalized hamming distance, W_j is the weight of each criterion

Completely unknown criteria weights:
The weights are calculated per the following non-linear programming model:

$$\begin{cases} \boldsymbol{max\, H\,(w)} = \frac{1}{3}\sum_{j=1}^{m}\sum_{i=1}^{m}\sum_{r=1}^{m} W_j(|T_{ij} - T_{rj}| + |I_{ij} - I_{rj}| + |F_{ij} - F_{rj}|) \\ \qquad\qquad\qquad s.t. \\ \qquad\qquad W_j \geq 0, \sum_{j=1}^{n} W_j^2 = 1, j = 1,2,\dots n \end{cases} \quad (10)$$

4.3 The third proposed technique (using the optimization method)

This approach is divided into two sub-approaches. The first one is to build an optimization model for calculating the unknown criteria weights either partially known or completely unknown depending on the TOPSIS technique. In the other one, the main concept of the VIKOR method is used to construct mathematical models for calculating the partially known or the completely unknown criteria weights.

The Neutrosophic TOPSIS:
Based on the main idea of the TOPSIS that is choosing the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). The optimization model can be constructed as a multi-objective model, in which the distance from the PIS (d^+) is minimized and the distance from the NIS (d^-) is maximized. Firstly, the absolute values of the SVN-PIS (a^+) and the SVN-NIS (a^-) are calculated which are (1, 1, 1) and (0, 0, 0) respectively. Then the criteria weights are calculated according to the linear programming model in (11) for the partially known criteria case. A non-linear programming model constructed in (12) is used for the completely unknown criteria case.

$$\begin{cases} mind^+ = \sum_{j=1}^{n} d(a_{ij}, a_j^+)W_j \\ \max d^- = \sum_{j=1}^{n} d(a_{ij}, a_j^-)W_j \\ s.t.\, W_j \geq 0, \sum_{j=1}^{n} W_j = 1, j = 1,2,\dots,n \end{cases} \quad (11)$$

$$\begin{cases} mind^+ = \sum_{j=1}^{n} d(a_{ij}, a_j^+)W_j \\ \max d^- = \sum_{j=1}^{n} d(a_{ij}, a_j^-)W_j \\ s.t.\, W_j \geq 0, \sum_{j=1}^{n} W_j^2 = 1, j = 1,2,\dots,n \end{cases} \quad (12)$$

The Neutrosophic VIKOR:
The main concept of the VIKOR method in ranking the alternatives depends on finding a compromise solution with the maximum group utility and the minimum individual regret value.

The optimization models used minimizes the S_i value which represents the separation measure of the alternative from the best value and maximize the R_i value which represents the separation measure of the alternative from the worst value. The weights are calculated based on the linear programming model in (13) for the partially known criteria weights and the non-linear programming model in (14) for the completely unknown criteria weights.

$$\begin{cases} minS_i \\ \max R_i \\ s.t.\, W_j \geq 0, \sum_{j=1}^{n} W_j = 1, j = 1,2,\dots,n \end{cases} \quad (13)$$

$$\begin{cases} minS_i \\ \max R_i \\ s.t.\, W_j \geq 0, \sum_{j=1}^{n} W_j^2 = 1, j = 1,2,\dots,n \end{cases} \quad (14)$$

5 NUMERICAL EXAMPLE

Consider the problem taken from [24]. After the pre-evaluation process, four suppliers have remained as alternatives for further evaluation, $A = (A_1, A_2, A_3, A_4)$. Four decision makers $DM = (DM_1, DM_2, DM_3, DM_4)$ are responsible to evaluate the alternatives. Their weights are $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0.25, 0.25, 0.25, 0.25)$ For the evaluation process, four different criteria have been selected by the company: $C = (C_1, C_2, C_3, C_4)$. SVN numbers are used to construct the decision matrices as illustrated in Tables 1, 2, 3, 4.

TABLE 1
DECISION MATRIX GIVEN BY DM_1

	C_1	C_2	C_3	C_4
A_1	(0.4, 0.3, 0.2)	(0.4, 0.2, 0.3)	(0.2, 0.2, 0.5)	(0.7, 0.2, 0.3)
A_2	(0.6, 0.1, 0.2)	(0.6, 0.1, 0.2)	(0.5, 0.2, 0.3)	(0.5, 0.1, 0.2)
A_3	(0.3, 0.2, 0.3)	(0.5, 0.2, 0.3)	(0.1, 0.5, 0.2)	(0.1, 0.4, 0.5)
A_4	(0.7, 0.2, 0.1)	(0.6, 0.1, 0.2)	(0.4, 0.3, 0.2)	(0.4, 0.5, 0.1)

TABLE 2
DECISION MATRIX GIVEN BY DM_2

	C_1	C_2	C_3	C_4
A_1	(0.1, 0.3, 0.5)	(0.5, 0.1, 0.5)	(0.3, 0.1, 0.6)	(0.4, 0.1, 0.4)
A_2	(0.2, 0.5, 0.4)	(0.3, 0.4, 0.3)	(0.2, 0.3, 0.1)	(0.2, 0.3, 0.5)
A_3	(0.5, 0.2, 0.6)	(0.2, 0.4, 0.3)	(0.5, 0.2, 0.5)	(0.1, 0.5, 0.3)
A_4	(0.2, 0.4, 0.2)	(0.1, 0.1, 0.3)	(0.1, 0.5, 0.4)	(0.5, 0.3, 0.1)

TABLE 3
DECISION MATRIX GIVEN BY DM_3

	C_1	C_2	C_3	C_4
A_1	(0.3, 0.2, 0.1)	(0.3, 0.1, 0.3)	(0.1, 0.4, 0.5)	(0.2, 0.3, 0.5)
A_2	(0.6, 0.1, 0.4)	(0.6, 0.4, 0.2)	(0.5, 0.4, 0.1)	(0.5, 0.2, 0.4)
A_3	(0.3, 0.3, 0.6)	(0.4, 0.2, 0.4)	(0.2, 0.3, 0.2)	(0.3, 0.5, 0.1)
A_4	(0.3, 0.6, 0.1)	(0.5, 0.3, 0.2)	(0.3, 0.3, 0.6)	(0.4, 0.3, 0.2)

TABLE 4
DECISION MATRIX GIVEN BY DM_4

	C_1	C_2	C_3	C_4
A_1	(0.2, 0.2, 0.3)	(0.3, 0.2, 0.3)	(0.2, 0.3, 0.5)	(0.4, 0.2, 0.5)
A_2	(0.4, 0.1, 0.2)	(0.6, 0.3, 0.5)	(0.1, 0.2, 0.2)	(0.5, 0.1, 0.2)
A_3	(0.3, 0.5, 0.1)	(0.2, 0.2, 0.3)	(0.5, 0.4, 0.3)	(0.5, 0.3, 0.2)
A_4	(0.3, 0.1, 0.1)	(0.2, 0.1, 0.4)	(0.2, 0.3, 0.2)	(0.3, 0.1, 0.6)

Firstly, the four decision matrices are aggregated into a single decision matrix using (6) as illustrated in Table 5.

TABLE 5
THE AGGREGATED DECISION MATRIX

	C_1	C_2	C_3	C_4
A_1	(0.2584, 0.2213, 0.2590)	(0.3808, 0.1414, 0.3409)	(0.2031, 0.2213, 0.5233)	(0.4578, 0.1861, 0.4162)
A_2	(0.4736, 0.1495, 0.2828)	(0.5399, 0.2632, 0.2783)	(0.3486, 0.2632, 0.1565)	(0.4377, 0.1565, 0.2991)
A_3	(0.3565, 0.2783, 0.3224)	(0.3380, 0.2378, 0.3224)	(0.3486, 0.3310, 0.2783)	(0.2703, 0.4162, 0.2340)
A_4	(0.4144, 0.2632, 0.1189)	(0.3840, 0.1316, 0.2632)	(0.2584, 0.3409, 0.3130)	(0.4042, 0.2590, 0.1861)

Then, we apply the proposed techniques to obtain the optimal solution and rank the alternatives.

Case 1: Assume that the criteria weights are partially known

The known information about the weights are:

$$w_1 \geq 0.18 \qquad w_1 \leq 0.20$$
$$w_2 \geq 0.15 \qquad w_2 \leq 0.25$$
$$w_3 \geq 0.30 \qquad w_3 \leq 0.35$$
$$w_4 \geq 0.30 \qquad w_4 \leq 0.40$$
$$w_j \geq 0, \sum_{j=1}^{n} w_j = 1, j = 1,2,3,4$$

Applying the proposed maximizing deviation method:

Using (9), the following linear programming model is constructed

$$max\, H(W) = 0.5886w_1 + 0.4590w_2 + 0.6961w_3 + 0.7344w_4$$
$$s.t.\, w_1 \geq 0.18 \qquad w_1 \leq 0.20$$
$$w_2 \geq 0.15 \qquad w_2 \leq 0.25$$
$$w_3 \geq 0.30 \qquad w_3 \leq 0.35$$
$$w_4 \geq 0.30 \qquad w_4 \leq 0.40$$
$$w_j \geq 0, \sum_{j=1}^{n} w_j = 1, j = 1,2,\dots,n$$

Solving this model using the simplex algorithm gives the weights as W = (0.18, .15, .30, 0.37). Then, by the Neutrosophic TOPSIS or the Neutrosophic VIKOR, the ranking is $A_2 > A_4 > A_1 > A_3$.

Applying the proposed optimization method:

Using the Neutrosophic TOPSIS method, the following linear programming model is constructed based on the absolute PIS and NIS using (11):

$$mind^+ = 1.4464w_1 + 1.4124w_2 + 1.4138w_3 + 1.3955w_4$$
$$maxd^- = 0.5981w_1 + 0.6399w_2 + 0.6242w_3 + 0.6555w_4$$
$$s.t.\, w_1 \geq 0.18 \qquad w_1 \leq 0.20$$
$$w_2 \geq 0.15 \qquad w_2 \leq 0.25$$
$$w_3 \geq 0.30 \qquad w_3 \leq 0.35$$
$$w_4 \geq 0.30 \qquad w_4 \leq 0.40$$
$$w_j \geq 0, \sum_{j=1}^{n} w_j = 1, j = 1,2,\dots,n$$

The criteria weights from this model are W = (0.18, .15, .30, 0.37), and the ranking order of the alternatives is $A_2 > A_4 > A_1 > A_3$.

Using the Neutrosophic VIKOR method, the following mathematical model is constructed by (13) as:

$$minS_i = 2.2163w_1 + 2.2292w_2 + 1.8231w_3 + 1.5416w_4$$
$$maxR_i = 0.8209w_1 + 0.8933W_2 + 0.8108w_3 + 0.7311W_4$$
$$s.t.\, w_1 \geq 0.18 \qquad w_1 \leq 0.20$$
$$w_2 \geq 0.15 \qquad w_2 \leq 0.25$$
$$w_3 \geq 0.30 \qquad w_3 \leq 0.35$$
$$w_4 \geq 0.30 \qquad w_4 \leq 0.40$$
$$w_j \geq 0, \sum_{j=1}^{n} w_j = 1, j = 1,2,\dots,n$$

Solving this model gives the same weights obtained by the Neutrosophic TOPSIS as W = (0.18, .15, .30, 0.37), and the ranking order of the alternatives is $A_2 > A_4 > A_1 > A_3$.

Case 2: Assume that the criteria weights are completely unknown

Applying the proposed entropy weights method:

After constructing the aggregated decision matrix, the entropy values are calculated using (7). These values are (0.6570, 0.5680, 0.7405, and 0.6440) for each criterion respectively. Then, the criteria weights are obtained using (8) as W = (.1315, .1656, .0994, .1364).

These weights are then applied with the Neutrosophic TOPSIS and the Neutrosophic VIKOR for ranking the alternatives. The ranking order from the Neutrosophic TOPSIS is $A_2 > A_4 > A_1 > A_3$ which means that the best alternative is the second one A_2. Applying the Neutrosophic VIKOR also gives the same ranking order.

Applying the proposed maximizing deviation method:

Constructing the following non-linear programming model using (10). By solving this model, the criteria weights are obtained as W = (0.4681, 0.3651, 0.5536, 0.5841).

$$max\, H(w) = 0.5886\, w_1 + 0.4590\, w_2 + 0.6961\, w_3 + 0.7344\, w_4$$
$$s.t. \sum_{j=1}^{n} w_j^2 = 1, w_j \geq 0$$

Applying the Neutrosophic TOPSIS, the ranking order is $A_2 > A_4 > A_1 > A_3$, while applying the Neutrosophic VIKOR, it gives the same ranking order as $A_2 > A_4 > A_1 > A_3$.

Applying the proposed optimization method:

Using the Neutrosophic TOPSIS method, the absolute PIS and NIS are calculated as (1, 1, 1) and (0, 0, 0) respectively. Then the following multi-objectives mathematical model is constructed using (12).

$$mind^+ = 1.4464W_1 + 1.4124W_2 + 1.4138\, W_3 + 1.3955W_4$$
$$maxd^- = 0.5981w_1 + 0.6399w_2 + 0.6242w_3 + 0.6555w_4$$
$$s.t. \sum_{j=1}^{n} w_j^2 = 1, w_j \geq 0$$

Solving this model using the weighted sum method with weight 0.5 for each objective, the criteria weights are W= (0.5379, 0.4898, 0.5006, 0.4692). By these weights, the ranking order from the Neutrosophic TOPSIS is $A_2 > A_4 > A_1 > A_3$

Using the Neutrosophic VIKOR method, the following multi-objectives model is constructed using (14)

$$minS_i = 2.2163w_1 + 2.2292w_2 + 1.8231w_3 + 1.5416w_4$$
$$maxR_i = 0.8209w_1 + 0.8933w_2 + 0.8108w_3 + 0.7311w_4$$
$$s.t. \sum_{j=1}^{n} w_j^2 = 1, w_j \geq 0$$

Solving this model, the criteria weights are W= (0.5997, 0.5742, 0.4351, 0.3484), and the ranking order is $A_2 > A_4 > A_1 > A_3$

the Neutrosophic VIKOR is much better than using the Neutrosophic TOPSIS from the time's point of view.

6 RESULTS AND DISCUSSION

To validate the proposed techniques, a comparison of the results is conducted under both the two cases, the partially known and the completely unknown criteria weights as mentioned in Table. 6. From this table, while solving the MCGDM problem, all techniques provided the same ranking order regardless of the different criteria weights as shown in the numerical example. In the partially known case, all the techniques give the same criteria weights with summation equals one. In the completely unknown case, despite the change in the weights obtained by each technique, all of them gives the same ranking order of the given alternatives and the same preferred alternative. The third technique is affected by the used method in solving the MCGDM problem. The criteria weights calculated by the optimization method under the TOPSIS is differ from those calculated under the VIKOR. Despite this difference in the calculated criteria weights, the ranking order of the alternatives is the same.

TABLE 6
THE COMPARISON OF THE CRITERIA WEIGHTS FOR THE DIFFERENT TECHNIQUES

| Proposed Techniques		Completely unknown	Partially known
Entropy weights		[.1315, .1656, .0994, .1364]	
Maximizing deviation		[0.4681,0.3651,.5536,0.5841]	[0.18, .15, .30, 0.37]
Optimization	TOPSIS	[0.5379,0.4898,0.5006,.4692]	[0.18, .15, .30, 0.37]
	VIKOR	[0.5997,0.5742,0.4351,0.3484]	[0.18, .15, .30, 0.37]

The calculation time of the proposed techniques with respect to the TOPSIS and the VIKOR has been investigated for both the partially known and the completely unknown cases. This study has been performed under three different perspectives, the change in the number of criteria, the number of alternatives and the number of decision makers. Four selected cases have been considered for each of those perspectives. For the completely unknown criteria weights, Fig. 2 (a), (b) presents the consumed time (in seconds) with respect to the change in the number of alternatives while solving by the Neutrosophic TOPSIS and the Neutrosophic VIKOR respectively. Similarly, Fig. 3 (a), (b) and Fig. 4 (a), (b) show the average calculation time for the change in both the number of criteria and the number of decision makers, respectively.

From these results, in the completely unknown case, the first proposed technique, which uses the entropy weights to calculate the criteria weights, outperforms the other two methods with respect to the time factor and the simplicity of calculations. Despite its superiority in saving the time, this technique lags the rule that the summation of the weights must be equal one. The usage of the maximizing deviation method with either the TOPSIS or the VIKOR gives the same ranking order with time less than of the optimization method. Nevertheless, the second technique depends mainly on the calculation of the hamming distance or the Euclidean distance, which means the possibility of differences in the calculated weights. However, the optimization method does not rely on any external calculations. Using
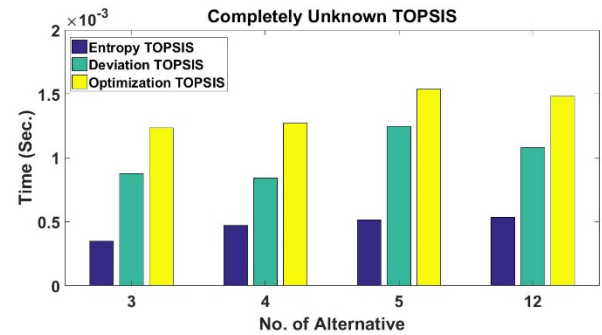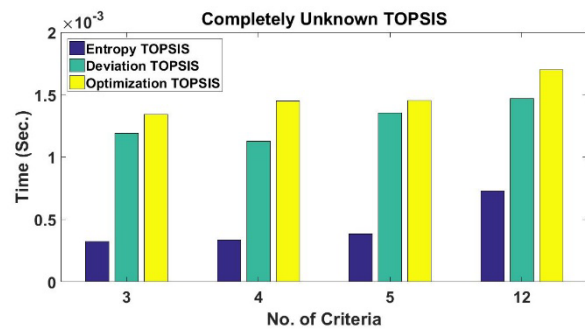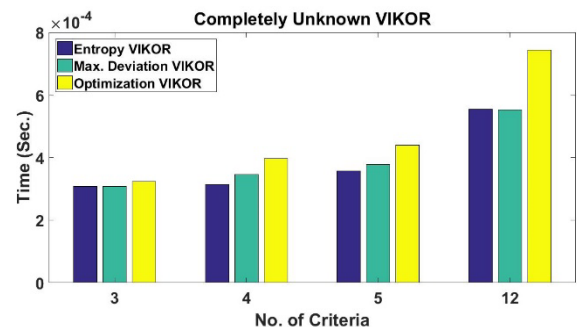

(a) Completely unknown TOPSIS
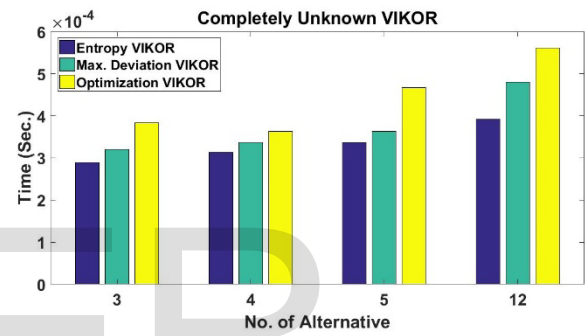
(b) Completely unknown VIKOR

Fig. 2: Average calculation time for change in the number of alternatives

(a) Completely unknown TOPSIS

(b) Completely unknown VIKOR

Fig. 3: Average calculation time for change in the number of criteria

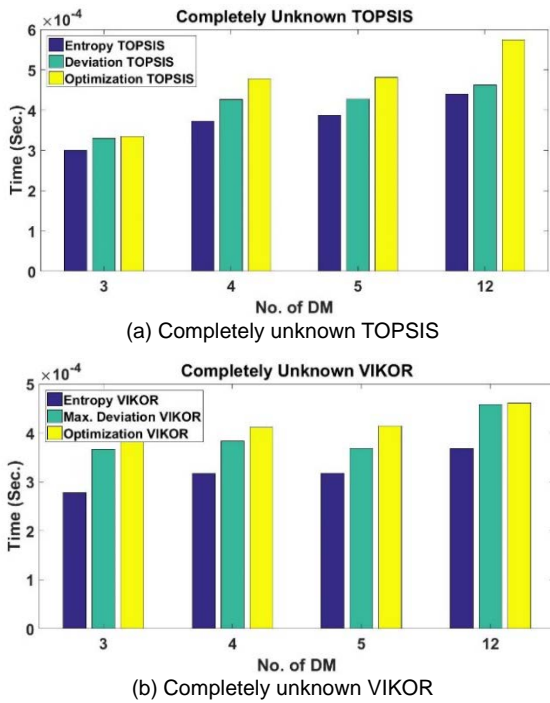(a) Completely unknown TOPSIS

(b) Completely unknown VIKOR

Fig. 4: Average calculation time for change in number of decision makers

For the partially known criteria weights, Fig. 5 (a), (b) presents the consumed time with the change in number of alternatives, while Fig. 6 (a), (b) and Fig. 7 (a), (b) are concerned to the change in number of criteria and decision makers, respectively.

In the partially known case, the two proposed techniques gave the same ranking order of the alternatives and the same criteria weights. Using the maximizing deviation method is recommended than the optimization method with the Neutrosophic VIKOR for ranking the alternatives with respect to the time factor.
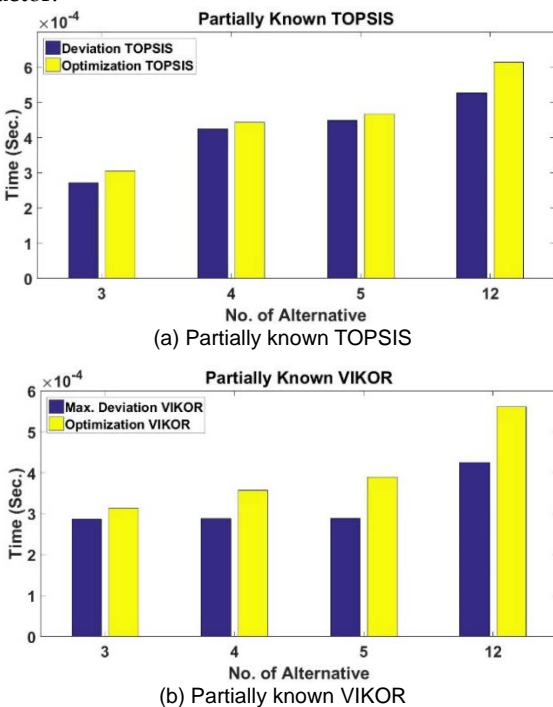

(a) Partially known TOPSIS

(b) Partially known VIKOR

Fig. 5: Average calculation time for change in the number of alternatives.

(a) Partially known TOPSIS

(b) Partially known VIKOR

Fig. 6: Average calculation time for change in the number of criteria.

(a) Partially known TOPSIS
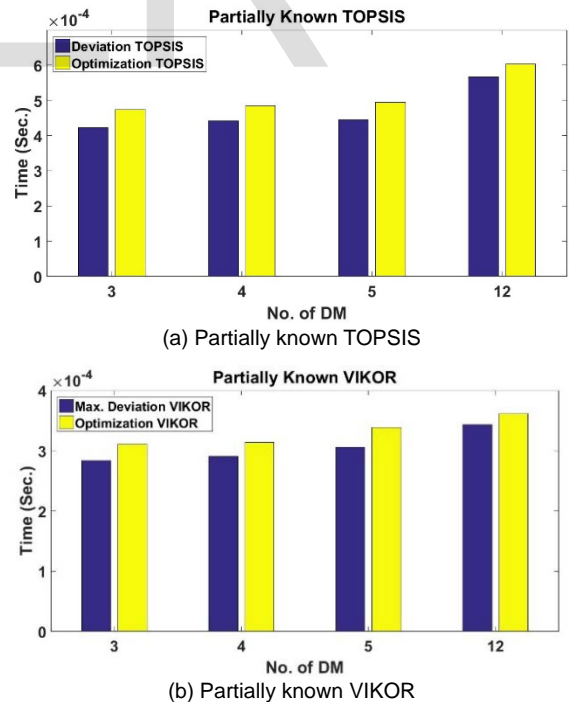

(b) Partially known VIKOR
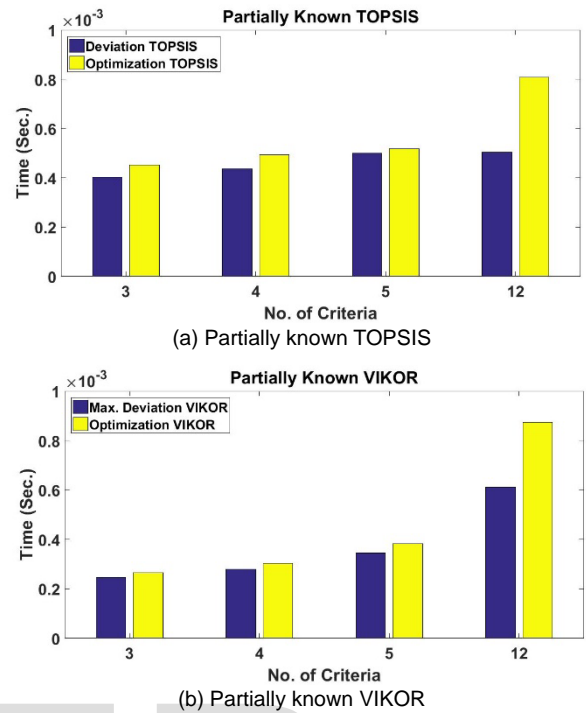
Fig. 7: Average calculation time for change in number of decision makers

7 CONCLUSIONS

In this paper, the major problem is to calculate the unknown criteria weights in the MCGDM problems. Three proposed techniques have been introduced for calculating these weights in either the completely unknown or the partially known forms. After the weights' calculation, they have been applied to the Neutrosophic TOPSIS and the Neutrosophic VIKOR methods for solving the Neutrosophic MCGDM problem and ranking the given alternatives. In the first technique, the entropy values are firstly calculated, and the completely unknown criteria weights are then calculated based on these values. In the second technique, the hamming distances are calculated for the aggregated decision matrix then, the maximizing deviation method is used to calculate the weights. A linear programming model is constructed to the partially known criteria weights but a non-linear programming model for the calculations in the completely unknown case. In the third proposed technique, based on the Neutrosophic TOPSIS, a linear programming model is constructed for the partially known criteria weights, while a non-linear programming model is constructed for calculating the completely unknown criteria weights. Another two mathematical models are constructed based on the Neutrosophic VIKOR for calculating the criteria weights. Then, these weights have been applied for solving the MCGDM problem and ranking the alternatives. The experimental results indicated that, the first technique is preferred in the computational process and saving time for the completely unknown case. While, the second and the third techniques are more appropriate for their accuracy in calculating the weights and the second is preferred in time. To validate the feasibility, useable, and practicality of the proposed technique, a brief comparison has been finally conducted. In the future work, the proposed techniques will be extended to deal with the interval-valued Neutrosophic sets.

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